Assignment 9

1. For each region described, use the indicated number of partitions to find the upper sum and the lower sum.

   (a) The area under \( y = 2^x \) over the interval \( 0 \leq x \leq 1 \), using \( n = 4 \) partitions.

   (b) The area under \( y = \ln x \) over the interval \( 1 \leq x \leq 4 \), using \( n = 3 \) partitions.

   (c) The area under \( y = \frac{1}{1 + x^2} \) over the interval \( -1 \leq x \leq 1 \), using \( n = 4 \) partitions.

   (d) The area under \( y = \sqrt{x} \) over the interval \( 1 \leq x \leq 4 \), using enough partitions to yield an error of less than 1 square unit.

2. Sketch the region indicated, then determine the value of the definite integral without using calculus.

   (a) \( \int_{-3}^{5} 8 \, dx \)

   (b) \( \int_{0}^{4} x \, dx \)

   (c) \( \int_{0}^{5} (10 - 2x) \, dx \)

   (d) \( \int_{1}^{3} x \, dx \)

   (e) \( \int_{2}^{2} \ln x \, dx \)

3. Express the following regions as definite integrals, then find upper and lower bounds for the area.

   (a) The region between the graph of \( y = x^3 \) and the \( x \)-axis, over the interval \( 0 \leq x \leq 3 \).

   (b) The region between the graphs of \( y = x \) and \( y = 12 - x^2 \).

   (c) The region between the graph of \( y = x^3 - 4x \) and the \( x \)-axis.
(d) The region between the graph of \( y = x^2 \) and the line \( y = 8 - 2x \) over the interval \(-5 \leq x \leq 5\).

(e) The region between the graph of \( y = \sin x \) and the \( x \)-axis, over the interval \( 0 \leq x \leq 2\pi \).

(f) The region bounded by the graph of \( x^2 + y^2 = 16 \).

4. Let \( \int_0^8 f(x) \, dx = 15 \), \( \int_0^8 f(x) \, dx = 20 \), and \( \int_0^8 g(x) \, dx = 16 \). Find the following.

(a) \( \int_5^8 f(x) \, dx \)

(b) \( \int_0^8 \left( f(x) + 3g(x) \right) \, dx \)

(c) \( \int_0^8 (4 + f(x)) \, dx \)

(d) \( \int_0^8 3f(x) \, dx \)

(e) Assume \( g(x) \) is increasing and \( g(5) = 2 \). Find an upper bound for \( \int_0^5 g(x) \, dx \).

(f) Assume \( f(x) \) is decreasing. Find an upper bound for \( f(5) \).

(g) Assume \( f(x) \) is decreasing. Find a lower bound for \( f(5) \).

5. Find the following.

(a) \( \lim_{x \to 5} \frac{x^2 - 12x + 35}{x^2 + 4x - 45} \)

(b) \( \lim_{x \to \infty} \frac{x^2 + 3x + 7}{5x^2 + 4x + 1} \)

(c) \( \lim_{x \to 1} \frac{6x^2 - 5x - 36}{x^2 - 7x + 6} \)

(d) \( \lim_{h \to 0} \frac{\sqrt{6 - 2h} - \sqrt{6}}{h} \)